ANALYSIS AND SYNTHESIS OF GUIDES WITH CLOSELY SPACED DISCONTINUITIES

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Abstract

Techniques have previously been developed to determine the scattering parameters for lines or guides with a single geometric step discontinuity ([1], [2], [7]). However, for a system with more than one closely spaced discontinuity the field interaction between the discontinuities must be taken into account. A technique outlined below, can in theory handle any number of closely spaced discontinuities.

Introduction

Microwave networks often contain abrupt changes in characteristic impedance of cylindrical or rectangular coaxial lines. It has been shown [1] that a lumped element can be used in a equivalent circuit to model such discontinuities to account for the energy in evanescent waves. The question arises whether this effect can be minimized by using several closely-spaced steps to achieve the change in characteristic impedance. The resulting reduction of the frequency dependence of the reflection coefficient at the junction between two lines of different characteristic impedance would improve the performance of many microwave networks.

The approach given here involves defining a matrix relating the modes at one side of a single discontinuity with the modes at the other side. An additional matrix is used to relate the modes at the output of a section of uniform guide to the modes at the input. These matrices can then be manipulated to obtain a relationship between the incident modes at one side of the region with multiple discontinuities and the transmit modes at the other side. This technique can be considered a generalization of the trasmission matrix analysis technique for single waves (see for example [6]) to include many higher order modes.

Theory

The development that follows will be based on the following assumptions:

1) The guide is a perfect conductor.
2) The coordinate z is parallel to the direction of propagation.
3) The discontinuities occur only in the z=constant plane.
4) Only a finite number of modes are needed to adequately specify the fields.

The transverse components of the fields can be written in general as

$$\overline{E}_t = \sum_{n=0}^{\infty} E_n \, \overline{e}_n \tag{1a}$$

$$\overline{H}_t = \sum_{n=0}^{\infty} H_n \, \hat{z} \times \overline{e}_n \tag{1b}$$

where

$$E_n = A_n e^{\gamma_n z} + A'_n e^{-\gamma_n z} \tag{2a}$$

$$H_n = Y_n (A_n e^{\gamma_n z} - A'_n e^{-\gamma_n z}) \tag{2b}$$

$$Y_n = -j\frac{\omega\varepsilon}{\gamma_n} \tag{2c}$$

At the discontinuity the following boundary conditions must be satisfied:

$$\overline{E}_{t,1} = \begin{cases} \overline{E}_{t,2} & \text{over } S_a \\ 0 & \text{over } S_c \end{cases} \tag{3a}$$

$$\overline{H}_{t,1} = \overline{H}_{t,2} \quad \text{over } S_a \tag{3b}$$

S_a refers to the aperture area and S_c refers to the conductor area at the discontinuity.

Using these conditions, the orthogonal properties of $\overline{e}_n$, and (1), the modes at one side of the discontinuity can be expressed as a linear combination of the modes on the other side. In matrix notation,

$$\mathbf{E}_1 = \mathbf{T}_e \mathbf{E}_2 \tag{4a}$$

$$\mathbf{T}_h \mathbf{H}_1 = \mathbf{H}_2 \tag{4b}$$

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where the matrix elements are

$$T_{e,nm} = \frac{\langle \bar{e}_{n,1}, \bar{e}_{m,2} \rangle_a}{\| \bar{e}_{n,1} \|^2_{a+c}}$$

$$T_{h,nm} = \frac{\langle \bar{e}_{n,2}, \bar{e}_{m,1} \rangle_a}{\| \bar{e}_{n,2} \|^2_a}$$

The inner product used is

$$\langle \bar{f}, \bar{g} \rangle = \int_S \bar{f} \cdot \bar{g}^* da$$

and the norm is given by

$$\| \bar{f} \|^2 = \langle \bar{f}, \bar{f} \rangle$$

It will be convenient to define a matrix $\mathbf{M}$ containing both the E-field and H-field modes such that

$$\mathbf{M} = \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}$$

Then (4) can be written as

$$\mathbf{M}_1 = \mathbf{D}\, \mathbf{M}_2$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{T}_e & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_h^{-1} \end{bmatrix}$$

From (2) the relationship between the modes in a region of guide with constant cross section can be found,

$$E(z_1) = \cosh[\gamma_n(z_2 - z_1)]E(z_2) + \gamma_n^{-1} \sinh[\gamma_n(z_2 - z_1)]H(z_2)$$

$$H(z_1) = \gamma_n \cosh[\gamma_n(z_2 - z_1)]E(z_2) - \sinh[\gamma_n(z_2 - z_1)]H(z_2)$$

In matrix notation,

$$\mathbf{M}(z_1) = \mathbf{LM}(z_2)$$

where

$$\mathbf{L} = \begin{bmatrix} \mathbf{C} & \mathbf{Y}^{-1}\mathbf{S} \\ \mathbf{YS} & \mathbf{C} \end{bmatrix}$$

$$C_{nm} = \begin{cases} \cosh[\gamma_n(z_2 - z_1)] & \text{if } n=m \\ 0 & \text{if } n \neq m \end{cases}$$

$$S_{nm} = \begin{cases} \sinh[\gamma_n(z_2 - z_1)] & \text{if } n=m \\ 0 & \text{if } n \neq m \end{cases}$$

$$Y_{nm} = \begin{cases} Y_n & \text{if } n=m \\ 0 & \text{if } n \neq m \end{cases}$$

With these tools at our disposal the relation between the modes in the incident region and the modes in the transmit region can easily be found. For example, if the discontinuity region consists of two discontinuities then we can write

$$\mathbf{M}_i = \mathbf{TM}_t$$

where $\mathbf{T} = \mathbf{D}_1 \mathbf{L} \mathbf{D}_2$

$\mathbf{D}_1$ is the discontinuity matrix for the first discontinuity, $\mathbf{L}$ is the line matrix representing the line between the discontinuities, and $\mathbf{D}_2$ is the discontinuity matrix for the second discontinuity.

If the first discontinuity the incident wave hits is taken to be at z=0, then from (2) the incident field at the first discontinuity can be written as

$$\mathbf{E}_t = \mathbf{A}_t + \mathbf{A}'_t$$

In addition, if $\mathbf{T}$ is partitioned such that

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{bmatrix}$$

then we can write

$$2\mathbf{Y}_i\mathbf{A} = [\mathbf{Y}_i(\mathbf{T}_{11} + \mathbf{Y}_t\mathbf{T}_{12}) + (\mathbf{T}_{21} + \mathbf{Y}_t\mathbf{T}_{22})]\, \mathbf{E}_t$$

and

$$2\mathbf{Y}_i\mathbf{A}' = [\mathbf{Y}_i(\mathbf{T}_{11} + \mathbf{Y}_t\mathbf{T}_{12}) - (\mathbf{T}_{21} + \mathbf{Y}_t\mathbf{T}_{22})]\, \mathbf{E}_t$$

from which the reflection coefficient and transmission coefficient can be found.

Parallel plate discontinuities

As an example the parallel plate case with an incident TEM wave is considered. For the parallel plate,

$$T_{e,00} = \alpha$$

$$T_{e,0m} = 0 \qquad m=1,2,\ldots$$

$$T_{e,n0} = \frac{2\sin(n\pi\alpha)}{n\pi} \qquad n=1,2,\ldots$$

$$T_{e,nm} = \frac{2n\sin(n\pi\alpha)(-1)^m}{\pi\,[n^2 - (m/\alpha)^2]} \qquad n,m=1,2,\ldots$$

$$T_{h,00} = 1$$

$$T_{h,0n} = \frac{\sin(n\pi\alpha)}{n\pi\alpha} \qquad n=1,2,\ldots$$

$$T_{h,m0} = 0 \qquad m=1,2,\ldots$$

$$T_{h,mn} = \frac{2n\sin(n\pi\alpha)(-1)^m}{\alpha\pi\,[n^2-(m/\alpha)^2]} \qquad n,m=1,2,\ldots$$

where

$$\alpha = \frac{h_2}{h_1}$$

h_1 = height of plate on left side of discontinuity

h_2 = height of plate on right side of discontinuity

A computer program was written to find the reflection coefficient and transmission coefficient for an arbitrarily shaped discontinuity region using the equations given above. The general orientation for the discontinuity region is shown in Figure 1. To validate this approach a comparison has been made between one case of treating each discontinuity separately using the equivalent capacitance technique and another case using the matrix technique outlined above. It would be expected that as the spacing between discontinuities increases, the two solutions should converge, since the evanescent waves created at one discontinuity will rapidly decay before reaching the next discontinuity. As shown in Figure 2 this is indeed the case.

An example showing an application of the above technique is shown in Figure 3; a frequency response of a tapered section with the height approximating a sinusoidal variation is given. In this example six modes were used in the calculations. As a further example a numerical optimization routine was used to find an optimal arrangement for an impedance match over a given bandwidth using a system with three discontinuities with the total discontinuity region being about a quarter wavelength long. Five modes were used in this example. This is shown in Figure 4.

Determining the number of modes to use

When using this approach care must be taken in picking the number of modes used in the calculations. The higher order modes will decay very rapidly between discontinuities. Modes of high enough order will be (numerically speaking) limited to the region around a discontinuity. To ask the computer to handle the coupling of these higher order modes can be quite a computational burden. It will be necessary to experiment with a different number of modes and observe the behavior of the scattering parameters and to observe the condition of the matrices used in the calculations. Of course, if the spacings are far enough apart then each discontinuity can be treated in isolation.

Conclusion

A technique has been outlined that accounts for the field interaction among closely spaced discontinuities along a guide. Examples were given to show how this technique can be used to approximate tapered sections and for computer optimization for a certain bandwidth about a given center frequency

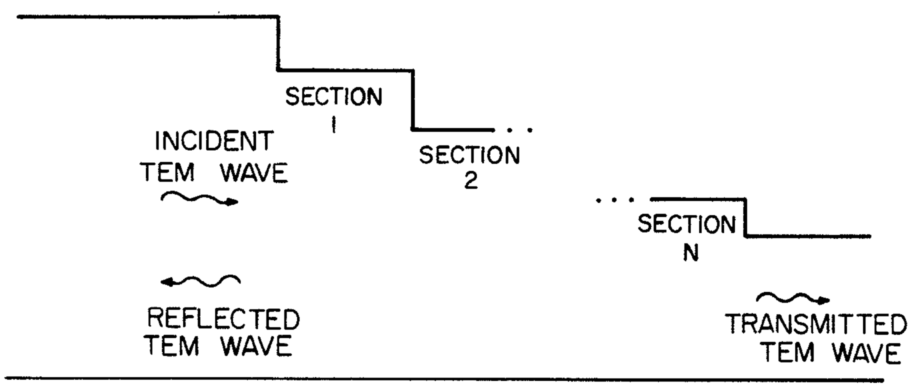

Figure 1. Cross section of parallel plate discontinuity region

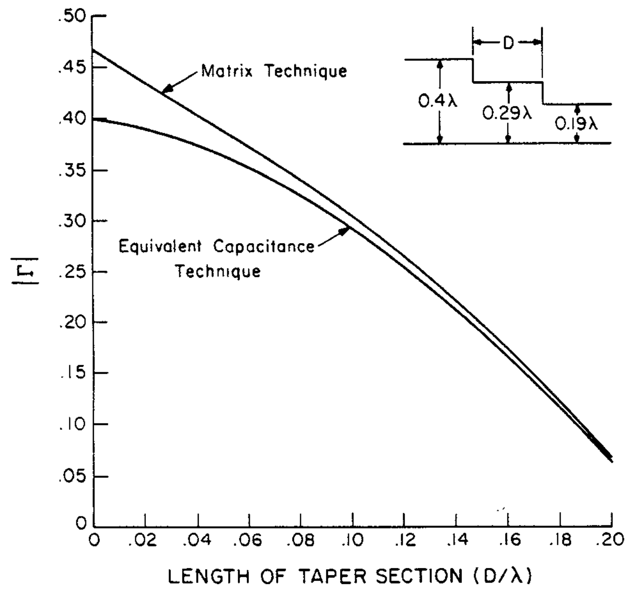

Figure 2. Comparision between matrix technique and capacitance technique in determining reflection coefficient as a function of mid-section length.

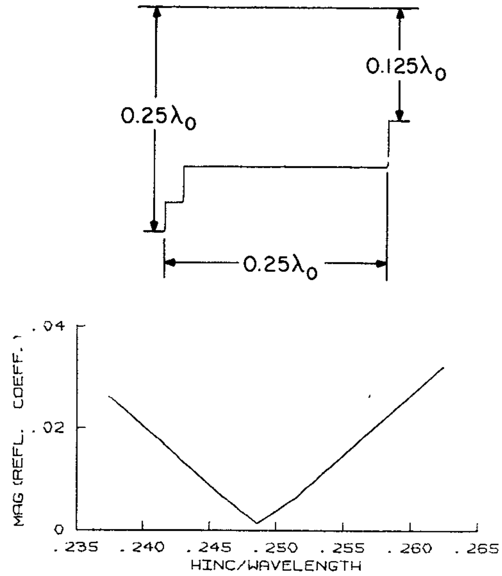

Figure 4. Example using an optimization routine to minimize the magnitude of the reflec. over a given bandwidth (.235< h_{inc}/λ <.265)

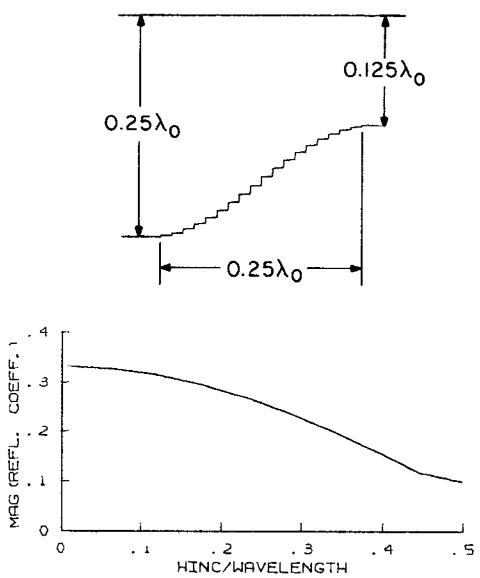

Figure 3. Example of technique applied to a taper section approximating a sinusoidal variation. λ_0 is the critical wavelength for which only one mode of propagation can occur if $\lambda < \lambda_0$

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